Lösningar till FMS012, 12/11-2007

1)

a) $0{,}90 = P(B|A) = \frac{P(A\cap B)}{P(A)} \Rightarrow P(A\cap B) = 0{,}36$

$0{,}94 = P(A\cup B) = P(A) + P(B) - P(A\cap B) \Rightarrow P(B) = 0{,}90$

$P(B|A) = P(B)$ $\therefore$ A och B är oberoende.

b) $P(X<3) = \sum_{j=0}^{2} P(X=j) = e^{-1{,}5}\left(1 + 1{,}5 + \frac{1{,}5^2}{2}\right)$

$\simeq 0{,}81$

c) $F_Y(y) = P(Y\le y) = P(1/X \le y) = P(X \ge 1/y)$

$= 1 - P(X \le 1/y) = 1 - 1/y \quad , y \ge 1$

$f_Y(y) = \frac{1}{y^2}$ för $y \ge 1$, annars $= 0$.

d) Sätt $P_k = P(X=k)$.

$\sum P_k = 1 \Rightarrow 1 = 1/4 + P_1 + P_2 + 1/5$ (1)

$E(X) = \sum k P_k \Rightarrow \frac{3}{2} = 0\cdot 1/4 + 1\cdot P_1 + 2\cdot P_2 + 3\cdot 1/5$ (2)

Ekvationerna (1) och (2) ger $P_1 = 1/5$ och $P_2 = 7/20$

2)

a) $P(X \leq 0,5) = \int_0^{0,5} \left(\int_0^x 8xy\,dy \right) dx = \int_0^{0,5} 4x^3\,dx = \frac{1}{16}$

b) $E(XY) = \int_0^1 \left(\int_0^x xy \cdot 8xy\,dy \right) dx = 8 \int_0^1 x^2 \left[\frac{x^3}{3} \right] dx$

$= 8 \left[\frac{x^6}{18} \right]_0^1 = \frac{8}{18}$

3)

a) $I_{\mu_2 - \mu_1} = \left(\bar{y} - \bar{x} \mp \lambda_{0,005} \cdot \sigma \cdot \sqrt{\frac{1}{n} + \frac{1}{m}} \right)$

$= \left(13,5 - 12,1 \mp 2,58 \cdot 0,6 \cdot \sqrt{\frac{1}{8} + \frac{1}{5}} \right)$

$\simeq (0,52 \quad 2,28) \quad (99\%)$

b)

längden av ett intervall med n obs

i varje stickprov =

$2 \cdot 2,58 \cdot 0,6 \sqrt{\frac{1}{n} + \frac{1}{n}} = 3,096 \sqrt{\frac{2}{n}}$

$3,096 \sqrt{\frac{2}{n}} \leq 0,4 \Rightarrow n > 120$

4)

a) $L(\lambda) = \prod_{i=1}^{n} 2\lambda x_i e^{-\lambda x_i^2} = (2\lambda)^n \cdot e^{-\lambda \sum x_i^2} \cdot \prod_{i=1}^{n} x_i$

$l(\lambda) = \ln L(\lambda) = n \ln(2\lambda) - \lambda \sum x_i^2 + \sum \ln x_i$

$$\frac{\partial l}{\partial \lambda} = \frac{n}{\lambda} - \sum x_i^2 = 0 \Rightarrow \lambda^* = \frac{n}{\sum x_i^2}$$

$$\lambda^*_{obs} = \frac{4}{\sum_{i=1}^{4} x_i^2} = \frac{4}{192{,}2105} \approx 0{,}0208$$

b) A = en komponent fungerar vid tiden 10

$$P(A) = \int_{10}^{\infty} 2\lambda x e^{-\lambda x^2} = e^{-100\lambda}$$

c) A = ena komp fungerar vid tiden 10
B = andra " " " " "

från b) får vi $P(A) = P(B) = e^{-100\lambda}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = P(A) + P(B) - P(A)P(B)$$

$$= 2e^{-100\lambda} - e^{-200\lambda}$$

d) $\lambda^*_{obs} = 0{,}0208 \Rightarrow P(A \cup B) = 0{,}234$

5)

a) $\beta^* = \frac{S_{xy}}{S_{xx}} = \frac{0{,}63}{0{,}94} = 0{,}67$

$$\alpha^* = \bar{y} - \beta^* \bar{x} = 2{,}8 - 0{,}67 \cdot 3 = 0{,}79$$

b) $$\beta^* = \frac{0{,}63 + \overbrace{(x_6 - 3)(y_i - \bar{y})}^{=0}}{0{,}94 + \underbrace{(x_6 - \bar{x})^2}_{=0}} = 0{,}67$$

$$\alpha^* = \frac{16,9}{6} - 0,673 = 0,8066$$

$\therefore$ β^* oförändrad, α^* ändras.

c)

$$V(\alpha^*) = \begin{cases} a)\ \sigma^2\left(\frac{1}{5} + \frac{3^2}{0,94}\right) \\ b)\ \sigma^2\left(\frac{1}{6} + \frac{3^2}{0,94}\right) \end{cases} \quad V(\beta^*) = \begin{cases} a)\ \sigma^2/0,94 \\ b)\ \sigma^2/0,94 \end{cases}$$

$\therefore$ α skattas bättre i b), skattning av β ej förbättras

6)

X_i = Livslängden hos A-lampor $\in N(100, 10)$

Y_i = ~ ~ B- ~ $\in N(80, 10)$

P(minst 4 byten under tiden 300)

= P(4 lampors livslängder ≤ 300)

$$= P(4\ A\text{-lampor}) \cdot P\left(\sum_{i=1}^{4} X_i \leq 300\right) +$$

$$+ P(3\ A\text{-lampor och } 1\ B) \cdot P\left(\sum_{i=1}^{3} X_i + Y_1 \leq 300\right)$$

$$+ \ldots + P(4\ B\text{-lampor}) P\left(\sum_{i=1}^{4} Y_i \leq 300\right)$$

$$= (0,6)^4 \Phi\left(\frac{300 - 4 \cdot 100}{\sqrt{400}}\right) + \binom{4}{3}(0,6)^3 0,4 \cdot \Phi\left(\frac{300-380}{\sqrt{400}}\right)$$

$$+ \binom{4}{2}(0,6)^2(0,4)^2 \Phi\left(\frac{300-360}{\sqrt{400}}\right) + \binom{4}{1} 0,6 \cdot (0,4)^3 \Phi\left(\frac{300-340}{\sqrt{400}}\right)$$

$$+ (0,4)^4 \Phi\left(\frac{300-320}{\sqrt{400}}\right) =$$

$$= 0{,}1296 \underbrace{\phi(-5)}_{0{,}0000} + 0{,}3456 \underbrace{\phi(-4)}_{0{,}0000} + 0{,}3456 \underbrace{\phi(-3)}_{0{,}0013}$$

$$+ 0{,}1536 \underbrace{\phi(-2)}_{0{,}0228} + 0{,}0256 \underbrace{\phi(-1)}_{0{,}1587} = 0{,}008$$