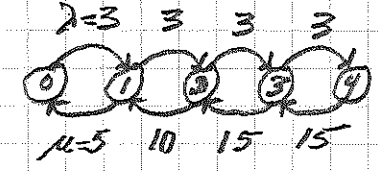
Svar/lösningar till tentamen i Kösystem (ETS075) 21/5-2012, 08-13.

1. a)



b) Standardberäkningar ger att,

$P_0 = \frac{1250}{2279}$, $P_1 = \frac{750}{2279}$, $P_2 = \frac{225}{2279}$,

$P_3 = \frac{45}{2279}$, $P_4 = \frac{9}{2279}$.

c)

$\lambda_{eff} \cdot 60 = \lambda(1-P_4)60 = 179.29$ per minut.

d)

$T = \frac{N}{\lambda_{eff}} = \frac{P_1 + 2P_2 + 3P_3 + 4P_4}{\lambda(1-P_4)} \approx 0.2013$ sekunder

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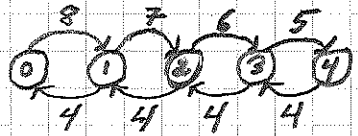
2. a)



, $\lambda_4 = 4$

Standardberäkningar ger att; $P_0 = 161/293$, $P_1 = 32/293$

$P_2 = 56/293$, $P_3 = 84/293$, $P_4 = 105/293$.

$1-P_0 \approx 0.9454 \Rightarrow \approx 94.5\%$ av tiden.

b) $P_0 + P_1 \approx 0.1638 \Rightarrow \approx 16.4\%$ av tiden.

c) $\lambda_{eff} = 8P_0 + 7P_1 + 6P_2 + 5P_3 \approx 3.782$

Anropssparr $= \frac{4P_4}{\lambda_{eff} + 4P_4} \approx 0.275$

d) $\lambda_4 P_4 \cdot 60 \approx 86$ spärrar/minut.

3

a) $\lambda_1 = 0.75\lambda + 0.2\lambda_1 \Rightarrow \lambda_1 = \frac{15}{16}\cdot\lambda = 15/4$

$\lambda_2 = 0.2\lambda_1 = 3/4$, $\lambda_3 = \lambda/4 = 1$

$T_1 = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{6 - \frac{15}{4}} = 4/9$

$T_2 = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{3 - \frac{3}{4}} = 4/9$

$T_3 = \frac{1}{\mu_3 - \lambda_3} = \frac{1}{2 - 1} = 1$, $T_4 = \frac{1}{\mu_4} = 1/8$

b) $E_2(\lambda_4/\mu_4) = E_2(4/8) = 0.0769$, dvs ca 7,7 %.

c) $W_{tot} = T_{tot} - \bar{X}_{tot}$

$T_{tot} = \frac{N_1 + N_2 + N_3 + N_4}{\lambda} = (\lambda_1 T_1 + \lambda_2 T_2 + \lambda_3 T_3 + \lambda_{4,eff} T_4)/\lambda =$

$= \left(\frac{15}{4}\cdot\frac{4}{9} + \frac{3}{4}\cdot\frac{4}{9} + 1 + 4(1 - 0.076923)\cdot\frac{1}{8}\right)/4 \approx 0.8654$

$\bar{X}_{tot} = (\lambda_1/\mu_1 + \lambda_2/\mu_2 + \lambda_3/\mu_3 + \lambda_{4,eff}/\mu_4)/4 =$

$= \left(15/24 + 3/12 + 1/2 + 4(1 - 0.076923)/8\right)/4 \approx 0.4591$

$\Rightarrow W_{tot} \approx 0.8654 - 0.4591 = \underline{\underline{0.4063}}$ (sek)

d) $\bar{X}_{tot} \approx 0.4591$ (sek), se c).

4

a) $N_i = \frac{\rho_i}{1-\rho_i}$

$\lambda_1 = 1 \Rightarrow \rho_1 = 1/4 \Rightarrow \underline{N_1 = 1/3}$

$\lambda_2 = 2 \Rightarrow \rho_2 = 1/2 \Rightarrow \underline{N_2 = 1}$

$\lambda_3 = \lambda_2 + 3\lambda_1/4 = 2.75 \Rightarrow \rho_3 \approx 0.458 \Rightarrow \underline{N_3 \approx 0.845}$

$\lambda_4 = 0.3\lambda_3 = 0.825 \Rightarrow \rho_4 = 0.165 \Rightarrow \underline{N_4 \approx 0.198}$

$\lambda_5 = 0.7\lambda_3 + 0.25\lambda_1 = 2.175 \Rightarrow \rho_5 = 0.435 \Rightarrow \underline{N_5 \approx 0.77}$

b) $T_i = N_i/\lambda_i$ vilket ger: $T_1 = 1/3$, $T_2 = 1/2$, $T_3 \approx 0.307$, $T_4 \approx 0.24$ och $T_5 \approx 0.354$.

Den sökta medeltiden är:

$$(T_2+T_3+T_5)\frac{0.7\lambda_2}{\lambda_5} + (T_1+T_3+T_5)\frac{0.75\lambda_1 0.7}{\lambda_5} + (T_1+T_5)\frac{0.25\lambda_1}{\lambda_5} \approx \underline{1.07 \text{ sek.}}$$

c) Som a) har vi även här tre möjliga "vägar."

Den sökta medeltiden är:

$$(T_1+T_3+T_4)\frac{0.75\lambda_1 0.3}{\lambda_1} + (T_1+T_3+T_5)\frac{0.75\lambda_1 0.7}{\lambda_1} + (T_1+T_5)\frac{0.25\lambda_1}{\lambda_1} \approx \underline{0.892 \text{ sek.}}$$

d) $\rho_1 = \frac{\lambda_1}{4} < 1 \Rightarrow \lambda_1 < 4$

$\rho_3 = \frac{2+3\lambda_1/4}{6} < 1 \Rightarrow \lambda_1 < 16/3$

$\rho_4 = \frac{0.3(2+3\lambda_1/4)}{5} < 1 \Rightarrow \lambda_1 < 17.6/0.9$

$\rho_5 = \frac{0.7(2+3\lambda_1/4)+0.25\lambda_1}{5} < 1 \Rightarrow \lambda_1 < \frac{3.6}{0.775}$

Nod 1 blir först instabil och det sker då $\lambda_1 = 4$.

5

a) $T = \frac{1}{\mu - \lambda} = \frac{1}{5 - \lambda}$

Kurvan går genom punkterna $(\lambda = 0,\ T = 0.2)$, $(\lambda = 2.5,\ T = 0.4)$, $(\lambda = 4,\ T = 1)$ och går asymptotiskt mot ∞ då $\lambda \to 5$.

b) $\lambda_{eff}/\mu = 1 - P_0 = \lambda(1 - P_{K+1})/\mu$

Personen har således rätt.

c) $A = \sum_{k=1}^{L} \mu_k P_k = \sum_{k=1}^{L} \lambda_{k-1} P_{k-1} = \sum_{k=0}^{L-1} \lambda_k P_k = \lambda_{eff}$

$$\text{Anropssannolikhet} = \frac{\lambda_L P_L}{A + \lambda_L P_L}$$

6

a) $T = \bar{x} + \frac{\lambda E\{x^2\}}{2(1 - \lambda \bar{x})} = 2.35 \cdot 10^{-3}$

ty $\bar{x} = 0.02$ och $E\{x^2\} = 0.02^2$

b) $W = \frac{\lambda E\{x^2\}}{2(1 - \lambda \bar{x})}$ med $\bar{x} = x_0 P_0 + x_1 (1 - P_0)$ och

$E\{x^2\} = x_0^2 P_0 + x_1^2 (1 - P_0)$

c) $N = \lambda T = \underbrace{\lambda W}_{N_q} + \underbrace{\lambda \bar{x}}_{N_s = S} \Rightarrow$ Rimligt.