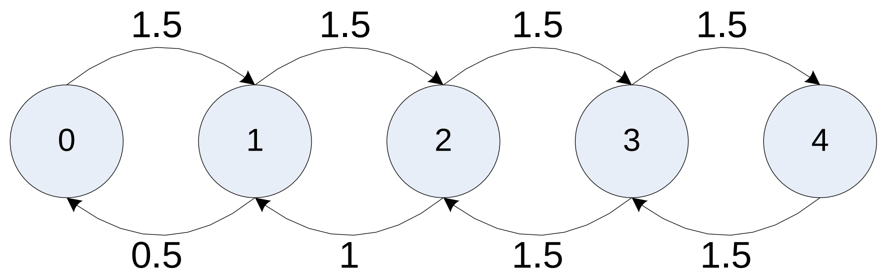
Lösningar till tentamen i Kösystem 4 Juni 2010

Problem 1.

a)



b) Snittmetoden ger följande samband: $p_1 = 3p_0, p_2 = \frac{9}{2}p_0, p_3 = \frac{9}{2}p_0, p_4 = \frac{9}{2}p_0$

Summan av alla tillståndssannolikheterna ska bli lika med 1 vilket ger att:

$p_0 = 2/35, p_1 = 6/35, p_2 = p_3 = p_4 = 9/35$

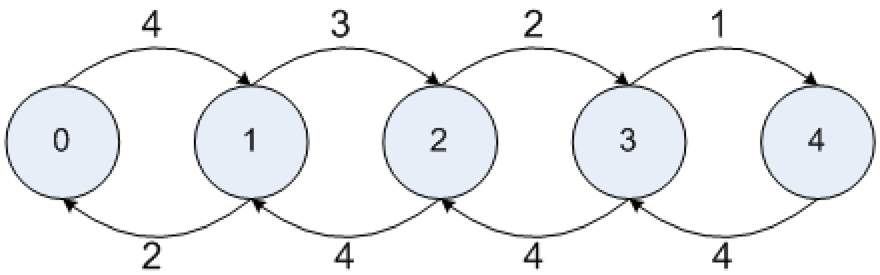
c) $60\lambda_{eff} = 60\lambda(1 - p_4) = 60\frac{3}{2}\left(1 - \frac{9}{35}\right) = 60\frac{39}{35} \approx 66.86$ jobb per minut.

d) Medelantalet jobb i kön, N_q ,beräknas här enklast som $N_q = p_4$.

Tillämpning av Littles sats ger svaret: $W = \frac{N_q}{\lambda_{eff}} = \frac{9/35}{39/35} = \frac{9}{39} \approx 0.231\ (sek)$

Problem 2.

a) $N_s = \lambda_{eff}/\mu$



Tillståndssannolikheterna beräknas enligt samma metod som i Problem 1, vilket ger:

$$p_0 = 16/87, p_1 = 32/87, p_2 = 24/87, p_3 = 12/87, p_4 = 3/87$$

$$N_s = \frac{\lambda_{eff}}{\mu} = \frac{1}{\mu}(4p_0 + 3p_1 + 2p_2 + p_3) = 110/87 \approx 1.26$$

b) Tidsspärren = $p_4 = 3/87 \approx 0.0345$
Anropsspärren = 0 ty $\lambda_4 = 0$

c) $p_2 + p_3 + p_4 = 13/29 \approx 0.45$, dvs 45% av tiden är bägge betjänarna upptagna i medel.

d) Se a).

Problem 3.

a) $T = \frac{1}{\mu(1-\rho)} = \frac{12.5}{\mu}$ vilket ger att $\rho = \frac{\lambda}{\mu} = 11.5/12.5 \approx 0.92$.

i) Om istället betjäningsintensiteten $\mu_1 = 2\mu$ används erhålles: $\rho_1 = \frac{\lambda}{\mu_1} = \frac{\lambda}{2\mu} = 11.5/25 \approx 0.46$

$$T_1 = \frac{1}{\mu_1(1-\rho_1)} = \frac{1}{2\mu(1-\rho_1)} = \frac{12.5/13.5}{\mu} \approx \frac{0.9259}{\mu} \approx 0.074T$$ (en väsentlig minskning!)

ii) $N = \frac{\rho}{1-\rho} = 11.5$ $\qquad N_1 = \frac{\rho_1}{1-\rho_1} = \frac{11.5}{13.5} \approx 0.852$

b) $N_q = \lambda W = \frac{\rho^2}{1-\rho} \leq 2$. Lösningen till detta är att $\rho \leq -1 + \sqrt{3} \approx 0.732$ vilket i sin tur medför att

$$T = \frac{1}{\mu(1-\rho)} \leq \frac{1}{\mu(2-\sqrt{3})} \approx \frac{3.73}{\mu}$$

c) $N_q = \frac{\rho^3}{4-\rho^2}$, $\qquad \rho = \lambda/\mu \approx 0.92$ (Rättelse: 3c) ska börja med texten: I a) fördubblades betjäningsintensiteten i ett …….)

i) $T = \frac{1}{\lambda}(N_q + \rho) = \frac{1}{\mu}\left(\frac{\rho^2}{4-\rho^2} + 1\right) \approx \frac{1.27}{\mu}$ $(sämre\ än\ ai))$

ii) M/M/2 och $\mu : N_q = \frac{\rho^3}{4-\rho^2} \approx 0.247$, $\qquad$ M/M/1 och $\mu_1 = 2\mu : N_q = \frac{\rho_1^2}{1-\rho_1} \approx 0.392$

iii) M/M/1 med dubblad betjäningsintensitet ger minst T.
M/M/2 med oförändrad betjäningsintensitet ger kortast kö i medel.

Problem 4.

a) $N_1 = \frac{\rho_1}{1-\rho_1} = \frac{0.75}{1-0.75} = 3$ $\qquad N_2 = \frac{\rho_2}{1-\rho_2} = \frac{0.8}{1-0.8} = 4$

$\lambda_4 = \lambda_1(1-\beta) + \lambda_2 = 17/4$ $\qquad N_4 = \frac{\rho_4}{1-\rho_4} = \frac{17/20}{1-17/20} = 17/3 \approx 5.67$

b) $T_1 = \frac{N_1}{\lambda_1} = 1, \qquad T_2 = \frac{N_2}{\lambda_2} = 2, \qquad T_4 = \frac{N_4}{\lambda_4} = 4/3$

i) $T_{ut\ nod\ 4} = \frac{\lambda_1(1-\beta)(T_1+T_4) + \lambda_2(T_2+T_4)}{\lambda_4}$ $\qquad T_{ut\ nod\ 4} = T_4 + \frac{9/4}{17/4}T_1 + \frac{2}{17/4}T_2 \approx 2.80$

ii) $W_1 = T_1 - \frac{1}{\mu_1} = 0.75, \quad W_2 = T_2 - \frac{1}{\mu_2} = 1.6, \quad W_4 = T_4 - \frac{1}{\mu_4} = \frac{17}{15}$

$W_{ut\ nod\ 4} = W_4 + \frac{9/4}{17/4}W_1 + \frac{2}{17/4}W_2 \approx 2.28$

c) Låt oss beteckna den sökta andelen med A: $A = \frac{\lambda_4}{\lambda_{eff,3} + \lambda_4}$

$\lambda_{eff,3} = \lambda_3\,(1 - p_{4097}) = \beta\lambda_1(1-y) = 0.75(1-y)$ $\qquad A = \frac{17/4}{0.75(1-y) + 17/4}$

d) $\lambda_{eff,3} = \beta\lambda_1(1-y)$ $\qquad N_{s,3} = \frac{\lambda_{eff,3}}{\mu_3} = \sum_{k=1}^{4097} p_k = 1 - x$ $\qquad \lambda_{eff,3} = \beta\lambda_1(1-y) = \mu_3(1-x)$

$\beta = \frac{4(1-x)}{3(1-y)}$

Problem 5.

a) $$T = \frac{N_1 + N_2 + N_3}{\lambda}, \quad N_i = \frac{\rho_i}{1-\rho_i}$$

$$\lambda_1 = \lambda + 0.3\lambda_3 \qquad \lambda_2 = 0.75\lambda_1 + 0.4\lambda_2 \qquad \lambda_3 = 0.7 \cdot 0.6\lambda_2$$

Ur dessa samband erhålles:

$$\lambda_2 = 0.75\lambda_1/0.6 \qquad \lambda_1 = \lambda + 0.3 \cdot 0.7 \cdot 0.6\lambda_2$$

$$\lambda_1 = \frac{\lambda}{0.8425} \approx 2.374 \qquad \lambda_2 = \frac{2.5}{0.8425} \approx 2.967 \qquad \lambda_3 = \frac{105}{84.25} \approx 1.246$$

$$N_1 = \frac{0.4748}{1-0.4748} \approx 0.9040 \qquad N_2 = \frac{0.2967}{1-0.2967} \approx 0.4219 \qquad N_3 = \frac{0.3739}{1-0.3739} \approx 0.5972$$

$$T = \frac{N_1 + N_2 + N_3}{\lambda} \approx 0.962 \quad (sek)$$

b) $$p_A = \frac{\lambda_A}{\lambda} = \frac{0.25\lambda_1}{\lambda} \approx 0.297 \qquad p_B = \frac{\lambda_B}{\lambda} = \frac{0.3 \cdot 0.6\lambda_2}{\lambda} \approx 0.267$$

c) $$T = p_A T_A + p_B T_B + p_C T_C \approx 0.962 \qquad p_C \approx 0.436$$

$$T_B = T_A + \frac{N_2}{0.75\lambda_1} \approx T_A + 0.237 \qquad T_C = T_A + \frac{N_2}{0.75\lambda_1} + \frac{N_3}{\lambda_3} \approx T_A + 0.716$$

$$T_A + p_B 0.237 + p_C 0.716 \approx 0.962 \qquad T_A \approx 0.587$$

$$T_C \approx 1.30$$

Problem 6.

$\lambda = 10, \quad E(X) = 0.05, \quad \rho = \lambda E(X) = 0.5 \qquad N = \rho + \frac{\lambda^2 E(X^2)}{2(1-\rho)} = 0.5 + 100E(X^2)$

a) $W = \frac{N}{\lambda} - E(X) = 10E(X^2) = 10 \cdot 0.05^2 = 0.025$

b)

$W = 10E(X^2) = 10(V(X) + E(X)^2) \geq 10E(X)^2 = 0.025$, dvs personen har rätt.

c)

$E(X^2) = \int_0^{0.1} x^2 \cdot 10dx = 1/300$

$W = 10E(X^2) = 1/30 \approx 0.033$